

Product cordial labeling of triple path union on C_3 , C_4 , C_5

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Abstract: A triple path union $P_m(3-G)$ is obtained by fusing three copies of same graph G at each vertex of path P_m . We take $G = C_3, FI(C_3)$, and $tail(C_3, 2p_2)$ to obtain $P_m(3-G)$ and show that all of them are product cordial graphs.

Key words: path union triangle, product cordial, labeling, fusion.

Subject Classification: 05C78

I. INTRODUCTION

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [9], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[11]. I.Cahit introduced the concept of cordial labeling [7]. There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [10] introduced the notion of **product cordial labeling**. A product cordial labeling of a graph G with vertex set V is a function f from V to $\{0,1\}$ such that if each edge uv is assigned the label $f(u)f(v)$, the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use $v_f(0,1) = (a, b)$ to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for $e_f(0,1) = (x, y)$.

A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gillian. We mention some part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; P_mUP_n ; C_mUP_n ; P_mUK1,n ; W_mUF_n (F_n is the fan P_n+K1); $K1,mUK1,n$; $W_mU K1,n$; W_mUP_n ; W_mUC_n ; the total graph of P_n (the total graph of P_n has vertex set $V(P_n) \cup E(P_n)$ with two vertices adjacent whenever they are neighbors in P_n); C_n if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n , provided t is even or both t and n are even; $K2+mK1$ if and only if m is odd; C_mUP_n if and only if $m+n$ is odd; $K_{m,n}UP_s$ if $s > mn$; C_n+2UK1,n ; $KnUKn,(n-1)/2$ when n is odd; $KnUKn-1,n/2$ when n is even; and $P2^n$ if and only if n is odd. are product cordial graphs. They also

prove that $K_{m,n}$ ($m,n > 2$), $P_m \times P_n$ ($m,n > 2$) and wheels are not product cordial and if a (p,q) -graph is product cordial graph, then $q = 6(p-1)(p+1)/4 + 1$. In this paper we show that $P_m(G)$ where Graph G is from $\{C_3, FI(C_3)$, and $tail(C_3, 2p_2)\}$ are product cordial graphs.

II. PRELIMINARIES

Fusion of vertex. Let G be a (p,q) graph. Let $u \neq v$ be two vertices of G . We replace them with single vertex w and all edges incident with u and that with v are made incident with w . If a loop is formed is deleted. The new graph has $p-1$ vertices and at least $q-1$ edges. If $u \in G_1$ and $v \in G_2$, where G_1 is (p_1, q_1) and G_2 is (p_2, q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with the concept is well elaborated in John Clark, Holton [6].

Path union of G , i.e. $P_m(G)$ is obtained by taking a path P_m and take m copies of graph G . Then fuse a copy each of G at every vertex of path at given fixed point on G . It has mp vertices and $mq + m-1$ edges. Where G is a (p,q) graph.

III. RESULTS

Theorem 1. $G = P_m(3-C_3)$ is product cordial for all m .

Proof: Take a path $P_m = (v_1, e_1, v_2, e_2, \dots, v_m)$. At i^{th} vertex of P_m fuse three copies of C_3 and are given by $u_{i,1} = v_i, u_{i,2}, u_{i,3}, u_{i,4}, u_{i,5}, u_{i,6}, u_{i,7}$. The vertex $u_{i,1}$ is common to path P_m and all three copies of C_3 . $|V(G)| = 7m$ and $|E(G)| = 10m-1$. Define a function $f : V(G) \rightarrow \{0,1\}$ as follows:

Case $m = 2x$

$f(u_{i,j}) = 0$ for all $i = 1, 2, \dots, x, j = 1, 2, \dots, 7$

$f(u_{i,j}) = 1$ for all $i = x+1, x+2, \dots, 2x; j = 1, 2, \dots, 7$.

The label number distribution is $v_f(0,1) = (7x, 7x)$ and $e_f(0,1) = (10x, 10x-1)$

Case $m = 2x+1$

To obtain a labeled copy of $P_{2x+1}(C_3)$ we label $P_{2x}(C_3)$ part of it from one end as given above. Further $f(u_{i,j}) = 1$ for $j = 1, 2, 3, 4$ and $i = 2x+1$;

$f(u_{i,j}) = 0$ for $j = 5, 6, 7$ and $i = 2x+1$. The label number distribution is $v_f(0,1) = (7x+3, 7x+4)$ and $e_f(0,1) = (10x+5, 10x+4)$.

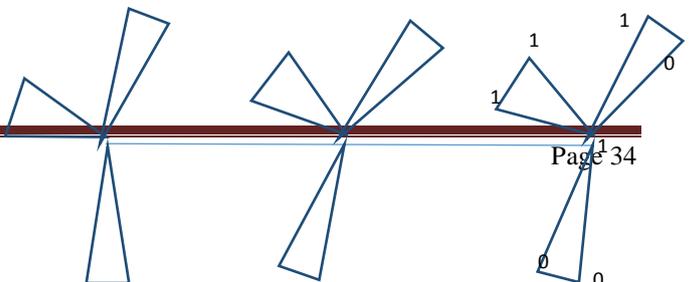


Fig 1 labeled copy

$$f(u_{i,j}) = 1 \text{ for all } i = x+1, x+2, \dots, 2x; j = 1, 2, \dots, 10.$$

The label number distribution is $v_f(0,1) = (10x, 10x)$ and $e_f(0,1) = (13x, 13x-1)$.

Case $m = 2x+1$

Thus the graph G is product cordial for all m .

Theorem 2 $G = P_m(3-G')$ is product cordial for all m where $G' = FL(C_3)$. (For all structures)

Proof: Take a path $P_m = (v_1, e_1, v_2, e_2, \dots, v_m)$. At i^{th} vertex of P_m fuse three copies of G' and are given by $u_{i,1}=v_i, u_{i,2}, u_{i,3}, u_{i,4}, u_{i,5}, u_{i,6}, u_{i,7}, u_{i,8}, u_{i,9}, u_{i,10}$ where $u_{i,4}, u_{i,7}$ and $u_{i,10}$ are pendent vertices. The vertex $u_{i,1}$ is common to path P_m and all three copies of C_3 . $|V(G)| = 10m$ and $|E(G)| = 13m-1$. Define a function $f: V(G) \rightarrow \{0,1\}$ as follows:

Structure I: This is obtained when two degree vertex on each of three copies of C_3 are fused with path vertex.

Case $m = 2x$

$$f(u_{i,j}) = 0 \text{ for all } i = 1, 2, \dots, x, j = 1, 2, \dots, 10$$

$$f(u_{i,j}) = 1 \text{ for all } i = x+1, x+2, \dots, 2x; j = 1, 2, \dots, 10.$$

The label number distribution is $v_f(0,1) = (10x, 10x)$ and $e_f(0,1) = (13x, 13x-1)$

Case $m = 2x+1$

To obtain a labeled copy of $P_{2x+1}(G')$ we label $P_{2x}(G')$ part of it from one end as given above with only two change given by

$$f(u_{i,4}) = 0 \text{ for } i = x+1;$$

$$f(u_{i,7}) = 0 \text{ for } i = x+1$$

$$\text{Further } f(u_{i,j}) = 1 \text{ for } j = 1, 2, \dots, 7 \text{ and } i = 2x+1;$$

$$f(u_{i,j}) = 0 \text{ for } j = 8, 9, 10 \text{ and } i = 2x+1.$$

All vertices labeled as

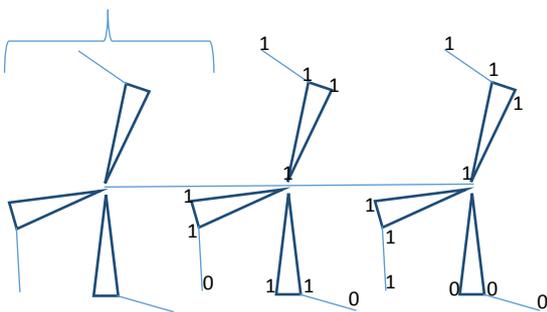


Fig .2 $P_3(FL(C_3))$: structure 1: $v_f(0,1) = (15, 15)$ and $e_f(0,1) = (13x+6, 13x+6)$

The label number distribution is $v_f(0,1) = (10x+5, 10x+5)$ and $e_f(0,1) = (13x+6, 13x+6)$.

Structure II: This is obtained when three degree vertex on each of three copies of C_3 are fused with path vertex.

Case $m = 2x$.

$$f(u_{i,j}) = 0 \text{ for all } i = 1, 2, \dots, x, j = 1, 2, \dots, 10$$

All vertices labeled as '0'

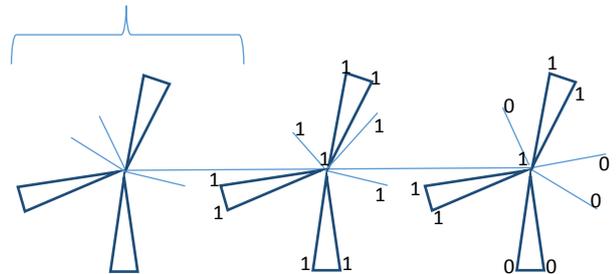


Fig 3 $P_3(FL(C_3))$: structure 2: $v_f(0,1) = (15, 15)$ and $e_f(0,1) = (13x, 13x-1)$

To obtain a labeled copy of $P_{2x+1}(G')$ we label $P_{2x}(G')$ part of it from one end as given above.

$$f(u_{i,j}) = 1 \text{ for } j = 1, 2, 3, 5, 6, \text{ and } i = 2x+1;$$

$$f(u_{i,j}) = 0 \text{ for } j = 4, 7, 8, 9, 10 \text{ and } i = 2x+1.$$

The label number distribution is $v_f(0,1) = (10x+5, 10x+5)$ and $e_f(0,1) = (13x+6, 13x+6)$.

Structure III: This is obtained when any of the pendent vertices on each of three copies of G' are fused with path vertex.

Case $m = 2x$.

$$f(u_{i,j}) = 0 \text{ for all } i = 1, 2, \dots, x, j = 1, 2, \dots, 10$$

$$f(u_{i,j}) = 1 \text{ for all } i = x+1, x+2, \dots, 2x; j = 1, 2, \dots, 10.$$

The label number distribution is $v_f(0,1) = (10x, 10x)$ and $e_f(0,1) = (13x, 13x-1)$.

For $m = 2x+1$ the product cordial labeling does not exist.

Theorem 3. $G = P_m(3-G')$ is product cordial for all m where $G' = \text{tail}(C_3, 2p_2)$

Proof: There are three structures on path union depending on which point on G' is used to obtain a path union by fusing with path vertex. In all structures a path $P_m = (v_1, e_1, v_2, e_2, \dots, v_m)$ remains unchanged. In **Structure I** At i^{th} vertex of P_m fuse three copies of G' and are given by $u_{i,1}=v_i, u_{i,2}, u_{i,3}, u_{i,4}, u_{i,5}, u_{i,6}, u_{i,7}, u_{i,8}, u_{i,9}, u_{i,10}, u_{i,11}, u_{i,12}, u_{i,13}$

where $u_{i,4}, u_{i,5}$ and $u_{i,8}, u_{i,9}, u_{i,12}, u_{i,13}$ are pendent vertices at $u_{i,3}, u_{i,3}, u_{i,7}, u_{i,7}, u_{i,11}, u_{i,11}$ respectively.

Define a function $f: V(G) \rightarrow \{0,1\}$ as follows:

Structure 1 is obtained when two degree vertex on each of three copies of G' are fused with path vertex. The vertex $u_{i,1}$ is common to path P_m and all three copies of G' . $|V(G)|= 13m$ and $|E(G)|= 16m-1$.

Case $m = 2x$

$$f(u_{i,j})= 0 \text{ for all } i = 1, 2, \dots, x, j = 1, 2, \dots, 13$$

$$f(u_{i,j}) = 1 \text{ for all } i = x+1, x+2, \dots, 2x; j = 1, 2, \dots, 13.$$

The label number distribution is $v_f(0,1) = (13x, 13x)$ and $e_f(0,1) = (16x, 16x-1)$

Case $m = 2x+1$

To obtain a labeled copy of $P_{2x+1}(G')$ we label $P_{2x}(G')$ part of it from one end as given above. Further $f(u_{i,j}) = 1$ for $j=1, 2, \dots, 6, 7$ and $i = 2x+1$;

$$f(u_{i,j}) = 0 \text{ for } j = 8, 9, \dots, 13 \text{ and } i = 2x+1.$$

The label number distribution is $v_f(0,1) = (13x+6, 13x+7)$ and $e_f(0,1) = (16x+7, 16x+8)$.

Structure II: This is obtained when four degree vertex on each of three copies of G' are fused with path vertex. At the i^{th} vertex of path P_m the design fused has ordinary label given as in the diagram below.

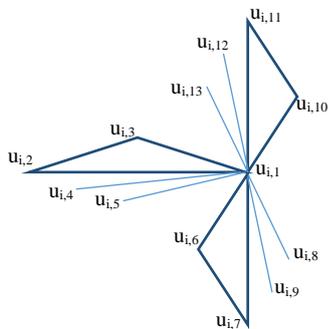


Fig 4 ordinary labeling of structure II fused at i^{th} vertex of P_m

Case $m = 2x$.

$$f(u_{i,j})= 0 \text{ for all } i = 1, 2, \dots, x, j = 5, 8, 9, 11, 12, 13$$

$$f(u_{i,j}) = 1 \text{ for all } i = x+1, x+2, \dots, 2x; j = 1, 2, 3, 4, 6, 7, 10$$

The label number distribution is $v_f(0,1) = (13x, 13x)$ and $e_f(0,1) = (16x, 16x-1)$.

Case $m = 2x+1$

To obtain a labeled copy of $P_{2x+1}(G')$ we label $P_{2x}(G')$ part of it from one end as given above.

$$\text{Further } f(u_{i,j}) = 0 \text{ for } j = 5, 8, 9, 11, 12, 13, i = 2x+1;$$

$$f(u_{i,j}) = 1 \text{ for } j = 1, 2, 3, 4, 6, 7, 10, 11 \text{ and } i = 2x+1.$$

The label number distribution is $v_f(0,1) = (13x+6, 13x+7)$ and $e_f(0,1) = (16x+7, 16x+8)$.

Structure III: This is obtained when any of the pendent vertices on each of three copies of G' are fused

i^{th} path vertex. At the i^{th} vertex of path P_m the design fused has ordinary label given as in the diagram below.

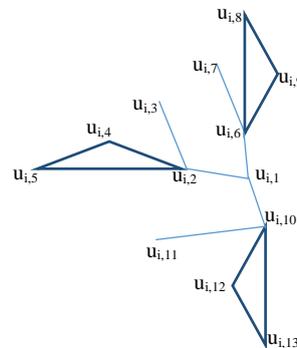


Fig .5 ordinary labeling of structure III fused at i^{th} vertex of P_m

Case $m = 2x$.

$$f(u_{i,j})= 0 \text{ for all } i = 1, 2, \dots, x, j = 1, 2, \dots, 13$$

$$f(u_{i,j}) = 1 \text{ for all } i = x+1, x+2, \dots, 2x; j = 1, 2, \dots, 13.$$

The label number distribution is $v_f(0,1) = (13x, 13x)$ and $e_f(0,1) = (16x, 16x-1)$.

Case $m = 2x+1$

To obtain a labeled copy of $P_{2x+1}(G')$ we label $P_{2x}(G')$ part of it from one end as given above.

$$\text{Further } f(u_{i,j}) = 0 \text{ for } j = 7, 8, 10, 11, 12, 13, i = 2x+1;$$

$$f(u_{i,j}) = 1 \text{ for } j = 1, 2, 3, 4, 5, 6, 9 \text{ and } i = 2x+1.$$

The label number distribution is $v_f(0,1) = (13x+6, 13x+7)$
and $e_f(0,1) = (16x+7, 16x+8)$.

IV. CONCLUSION

In this paper we have obtained path union by fusing three copies of same graph G at a particular vertex of G with path vertex. This is triple path union and is denoted by $P_m(3-G)$. We discuss resultant structure for product cordial labeling. We have proved that:

- 1) $P_m(3-C_3)$ is product cordial for all m .
- 2) $P_m(3-G')$ is product cordial for all m where $G' = FL(C_3)$. (For all structures)
- 3) $P_m(3-G')$ is product cordial for all m where $G' = \text{tail}(C_3, 2p2)$.

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